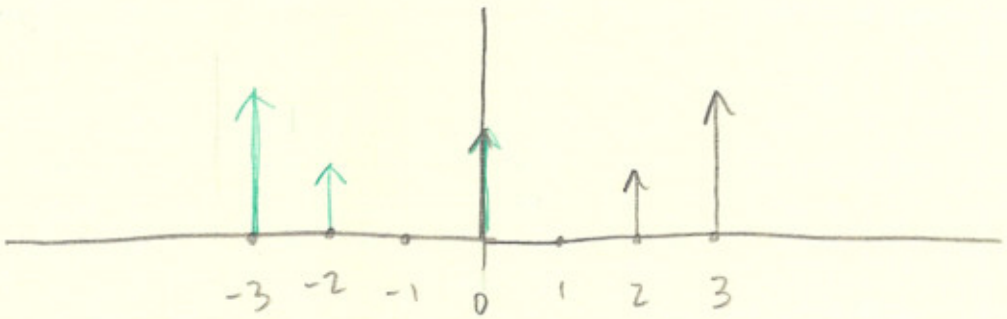
1.1.2 Operations on Sequences. (cont)

D) Folding

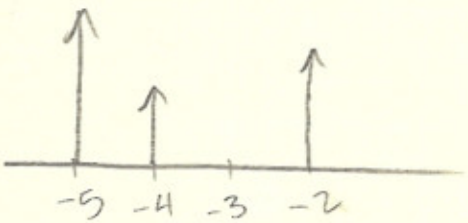
$$y[n] = x[-n]$$

$x[n]$ is flipped around $n=0$



Ex # shifting of a folded seq.

$x[-n-2]$



E) Energy and Power of a seq.

$$E_x = \sum_{n=-\infty}^{\infty} |x[n]|^2$$

Avg power of a seq $\in T = N$

$$P_x = \frac{1}{N} \sum_{n=0}^{N-1} |\tilde{x}[n]|^2$$

$\delta[n]$ $\qquad\qquad$ $U[n]$

$$\delta[n] = U[n] - U[n-1]$$

$$U[n] = \sum_{k=0}^{\infty} \delta[n-k]$$

1.1.3 Periodicy of a sine signal

$$x[n] = A\cos(\omega_0 n + \phi)$$

A sequence is periodic iff

$$x[n] = x[n+N]$$

$$\therefore \quad A\cos(\omega_0 n + \phi)$$

$$= A\cos(\omega_0 n + \omega_0 N$$

This is valid when

$$N = \frac{2\pi k}{\omega_0} \qquad k \in I$$

Ex $\omega_0 = \frac{3\pi}{4}$ rad/s

$$\frac{3\pi}{4} N = 2\pi K$$

$$K = \frac{3}{8} N$$

∴ fundamental freq is when $N = 8$.

Ex $\omega_0 = 1$ rad/s

$$K = \frac{N}{2\pi}$$

∴ there is no value of N that is an int that can make K an int.

Ex $x[n] = 2\cos(\pi/8 n + 15°)$

$$+ 5\sin(\pi/16 n + 30°)$$

$\omega_1 = \pi/8$ $\quad N = 16K$

$\omega_2 = \pi/16$ $\quad N = 32K$

∴ $N_{min} = 16$.

∴ $N_{min} = 32$

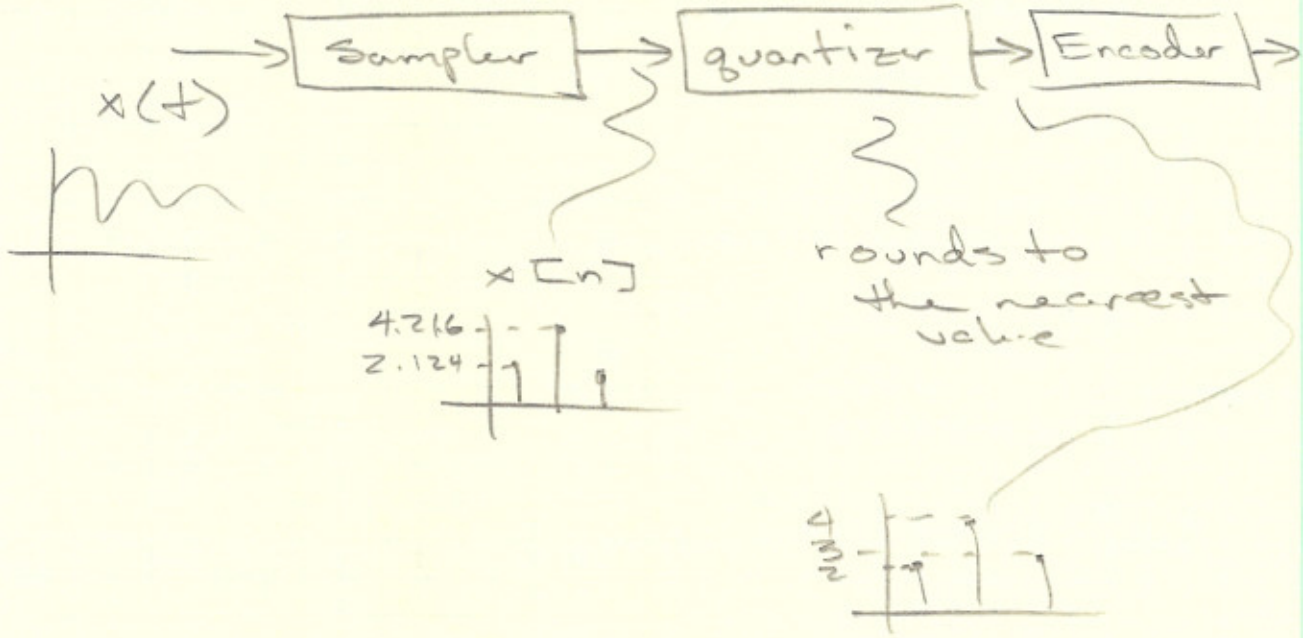
$T_0 = 32$

Ex $x[n] = 2\cos(\pi/8\, n + 15°) + 5\sin(\pi/16\, n + 30°) + \cos(\pi/6\, n) + e^{j(\pi/8 n)}$

$T_0 = 96$ (make sure to show work)

1.1.4 A/D – D/A Conversions

A/D:



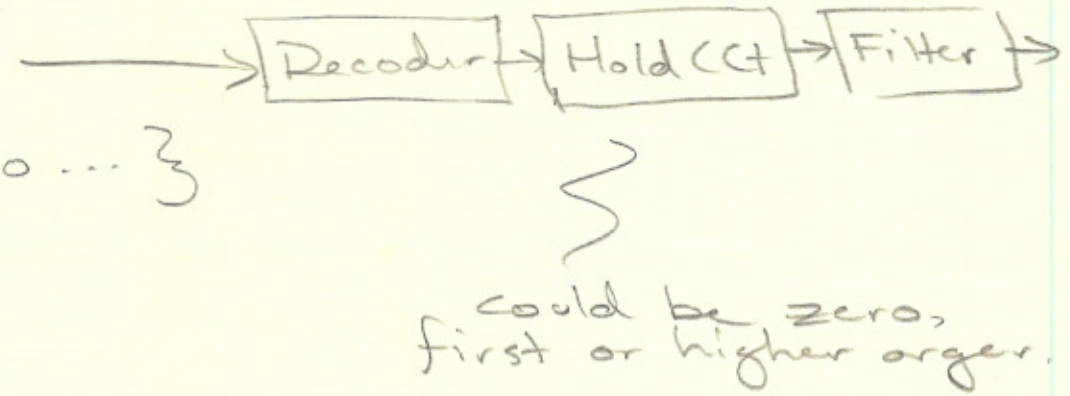
The rounding that occurs in the quantizer is called the quantization noise.

The value allowed in the digital signal is called the level of quantizer.

$$\Delta \text{ (step size)} = \frac{x_{max} - x_{min}}{L - 1}$$

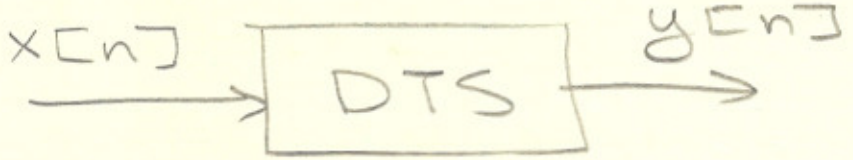
Where L is the number of levels. (do not forget that 0 is a level)

D/A:



1.2 Discrete time system (DTS)

A DTS is defined mathematically a an opperaterator that maps an input seq. $x[n]$ into an o/p seq. $y[n]$



Ex Ideal delay system

$$y[n] = x[n - n_d]$$

$$n_d \geq 0$$

EX Moving avg filter

$$y[n] = \frac{1}{3}(x[n] + x[n-1] + x[n-2])$$

In general

$$y[n] = \frac{1}{M_1 + M_2 + 1} \sum_{n=-M_1}^{M_2} x[n-k]$$

Sometimes called the moving avg filter if it is used to smooth out data.

1.2.1 Types of DTS

A) static or memoryless system

$$y[n] = f(x[n])$$

EX

$$y[n] = 3x[n] + 5(x[n])^2$$

B) Dynamic or Memory system.

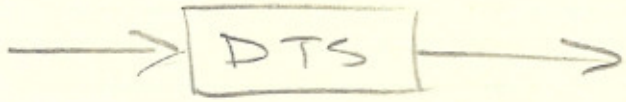
Past values of input affect current value

EX

$$y[n] = 3x[n] + 5x[n-2]$$

c) Linear System

for this system the principle of superposition is valid



$$x_1[n] \longrightarrow y_1[n]$$

$$a\,x_1[n] \longrightarrow a\,y_1[n]$$

$$x_1[n] + x_2[n] \rightarrow y_1[n] + y_2[n]$$

D) Non-Linear System

$$y[n] = (x[n])^2$$